

Move an Asteroid Competition - 2018

Orbital analysis of a reusable CubeSat platform to relay communications for rovers on an asteroidArunkumar Rathinam^{a*}^a Australian Centre for Space Engineering Research, School of Electrical Engineering and Telecommunications, UNSW Sydney, Australia, a.rathinam@unsw.edu.au

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Abstract

In recent exploration missions to asteroids, landers/rovers based on the CubeSat platform are sent to investigate in-situ the surface of an asteroid. With increasing interest in small celestial body exploration and in-situ resource utilization, the future missions will have a more complex system of landers/rovers and spacecraft. In a current generic architecture of an orbiting spacecraft and a rover, the direct communication link between them is severely limited for nearly half the rotational period of the asteroid. This work presents a conceptual approach, having an additional reusable CubeSat orbiting around the asteroid to relay communications to the rovers. This work analyzes typical orbit orbital combinations of periodic, quasi-periodic and terminator orbits, to solve for a close approach window with a minimum Δv between the trajectories of the master spacecraft and the CubeSat. The close approach orbital region is necessary for the master spacecraft to rendezvous, capture and dock the CubeSat platform. Numerical simulations were performed using the asteroid Castalia and a combination of orbital configurations for the master and CubeSat. We find that with both spacecraft in quasi-terminator orbits, the length of the rendezvous window is more consistent and the delta-v between the spacecraft is minimized at closest approach.

Keywords:(maximum 6 keywords)**Nomenclature**

μ_{Sun}	Gravitational parameter of Sun ($1.327 \times 10^{11} km^3/s^2$)
μ_{Ast}	Gravitational parameter of Asteroid
R	Constant distance b/w Sun and Asteroid
$G1$	Solar flux constant ($1 \times 10^{17} kgm^3/s^2/m^2$)
σ	Density (kg/m^3)
m/A	Mass to projected area of spacecraft (kg/m^2)
N	Mean motion
J	Jacobi integral
\mathbf{r}_i	Vector from variable field point to polyhedron vertex
\mathbf{E}_e	Edge dyad $\in \mathbb{R}^{3 \times 3}$
\mathbf{F}_f	Face dyad $\in \mathbb{R}^{3 \times 3}$
L_e	Factor per-edge $\in \mathbb{R}^1$
ω_f	Factor per-face $\in \mathbb{R}^1$
\mathbf{v}	Velocity vector $\in \mathbb{R}^{3 \times 1}$
$\boldsymbol{\omega}$	Angular velocity vector $\in \mathbb{R}^{3 \times 1}$
β	Non-dimensional acceleration due to SRP
U	Gravitational potential $\in \mathbb{R}^1$
∇U	Gravitational attraction $\in \mathbb{R}^{3 \times 1}$
$\nabla \nabla U$	Gravity gradient matrix $\in \mathbb{R}^{3 \times 3}$
$\nabla^2 U$	Laplacian $\in \mathbb{R}^1$

Acronyms/Abbreviations

SRP	Solar Radiation Pressure
ANH3BP	Augmented Normalized Hill Three Body Problem

1. Introduction

To date, asteroid exploration has been restricted to rendezvous, in-situ experiments, and sample-return missions. In recent years, mineral-rich asteroids became targets for space resource mining companies for in-situ resource extraction and utilization. The interest from private enterprises towards mining will boost future space exploration activities. This will increase the number of future missions and thus spacecraft are expected to perform autonomously and coordinate with other specialized crafts/landers [1] or purpose built rover platforms.

Experimental micro-platforms were used in previous exploration missions to conduct experiments on asteroid surfaces. MINERVA, a lander that was sent along with the first Hayabusa mission, unfortunately couldn't reach the asteroid because of an incorrect timing of deployment [2]. MASCOT Lander [3] (based on the CubeSat platform), a part of the

Hayabusa-2 mission, is to scout and perform scientific experiments on the asteroid's surface. The lander is relatively small (28cm x 29cm x 21cm weighs $\sim 9.8\text{kg}$) when compared to Philae [4] that landed on the Comet 67P/Churyumov-Gerasimenko. It is equipped with four scientific instruments (MASCAM, MicrOmega, MARA and MASMAG) [3] and has a limited operating time between 16 \sim 20 hours to explore the surface and collect useful scientific data. In another exciting development, the first deep space CubeSats Mars Cube One (MarCO) were launched along with NASA's Insight lander and it is the first ever CubeSat platform flown in deep space. If their journey is successful, MarCO (A & B) will relay data of InSight's entry, descent and landing back to Earth [5]. MarCO are based on a six unit CubeSat platform and the pair will carry their own communications and navigation experiments as they fly independently to Mars. The MASCOT-2 lander and two CubeSat Opportunity Payload Inter-satellite Network Sensors (COPINS) are also proposed for the future Asteroid Impact Mission mission [6]. Considering the progress of CubeSat technologies, future missions are expected to carry more sophisticated CubeSat payloads for reconnaissance purpose.

As interest in space resource mining grows, future missions are expected to carry and deploy purpose built rover/lander platforms on the asteroid's surface. With rotation periods varying between 6 \sim 12 hours for most NEAs [7] communications with a rover/lander will be severely limited. A generic asteroid mission architecture comprised of a master spacecraft and CubeSat communications relay to support a lander/rover could allow for constant communications, reducing technical risk and increasing capability. MarCOA & B will demonstrate that the CubeSat is a platform capable of delivering interplanetary communications. However, unlike Insight, missions to the asteroids will benefit from reusable CubeSat communication relays as they will then be able to visit multiple bodies.

This work presents the possibility of a reusable CubeSat to relay communication to the rover deployed on the asteroid surface. The operation architecture consists of a Master Spacecraft, a CubeSat (relay) platform and a rover on the asteroid surface. The CubeSat is assumed to be based on 6U platform and equipped with a communications payload to relay the data between the rover and miniature electric thrusters to perform orbit manoeuvres. The CubeSat's orbit is designed such that it frequently performs a flyby of the master spacecraft, thus providing

an opportunity for the master spacecraft to perform a minimal energy manoeuvre and capture the orbiting CubeSat using the robotic arm and dock them. This work analyzes possible stable orbits around asteroids having such operational configuration. Section [2] briefly discusses the dynamics around an asteroid, computing the gravitational potential and the equations of motion near small celestial bodies. Section [3] discusses the different orbits such as periodic orbits, quasi-periodic and terminator orbits and the cases for numerical simulation. The results are discussed based on the requirement of a close approach window with minimum Δv between the trajectories. Section [4] provides a brief discussion of the results and the future challenges.

2. Dynamics near an asteroid

The dynamics in the vicinity of an asteroid are extremely complex and the main factors that contribute include irregular shape, mass distribution, rotational period, solar perturbation (depending on asteroid's orbital position around the sun) and rare disturbances during close approach near the large planets.

2.1 Gravity potential

The gravitational attraction of the asteroid is the dominant factor affecting the close proximity orbits around an asteroid. The asteroid's shape and mass distribution play a critical role in generating a non-uniform gravity potential. Accurate computation of potential is necessary for close proximity trajectories to avoid collision with the asteroid's surface. There are different established methods available in the literature to calculate the gravitational potential of a non-spherical body. These include spherical harmonic expansion, polyhedral approaches and mass concentration, each method has its own advantage and disadvantages. The polyhedral approach is widely used to compute the gravity potential, attraction, gravity gradient and Laplace potential [8]. In order to compute an accurate gravitational potential we need a good shape model of the asteroid. Previous asteroid exploration missions adopted a *hovering* approach during proximity operations, it would be interesting for future missions to find potential stable orbits. The gravitational potential U , attraction ∇U , gradient matrix $\nabla\nabla U$ and the Laplacian $\nabla^2 U$ of a constant density polyhedron [8] is given below.

$$U = \frac{1}{2}G\sigma \sum_{e \in \text{edges}} \mathbf{r}_e \cdot \mathbf{E}_e \cdot \mathbf{r}_e \cdot L_e - \frac{1}{2}G\sigma \sum_{f \in \text{faces}} \mathbf{r}_f \cdot \mathbf{F}_f \cdot \mathbf{r}_f \cdot \omega_f \quad (1)$$

$$\nabla U = -G\sigma \sum_{e \in \text{edges}} \mathbf{E}_e \cdot \mathbf{r}_e \cdot L_e + G\sigma \sum_{f \in \text{faces}} \mathbf{F}_f \cdot \mathbf{r}_f \cdot \omega_f \quad (2)$$

$$\nabla \nabla U = G\sigma \sum_{e \in \text{edges}} \mathbf{E}_e \cdot L_e + G\sigma \sum_{f \in \text{faces}} \mathbf{F}_f \cdot \omega_f \quad (3)$$

$$\nabla^2 U = -G\sigma \sum_{f \in \text{faces}} \omega_f \quad (4)$$

In this work we consider the dynamics about asteroid 4769 Castalia using the shape model available in [9]. The shape model of the asteroid shown in fig. 1 consists of 4092 faces, 6138 edges and 2048 vertices. The density of the asteroid Castalia is 2.1 g.cm^{-3} and the rotational period is 4.07 hours [10]. The computed gravitational potential is shown in fig. 2.

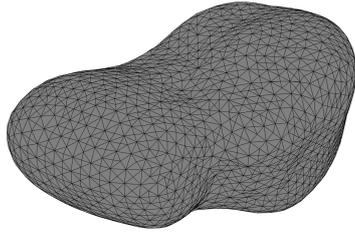


Fig. 1: 3D model of asteroid Castalia

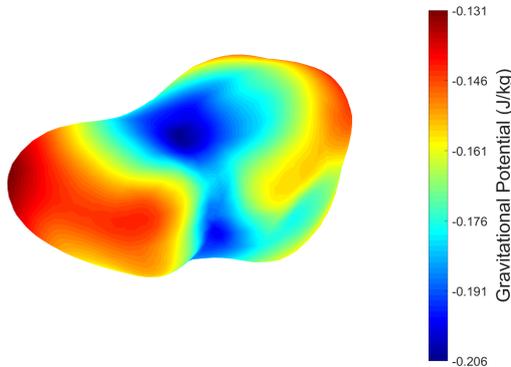


Fig. 2: Gravitational potential of Castalia

2.2 Equations of motion

The equations of motion of a spacecraft (including Coriolis and centripetal accelerations) defined in the body-fixed frame of an uniformly rotating asteroid are given by [10],

$$\ddot{\mathbf{r}} + 2\boldsymbol{\omega} \times \dot{\mathbf{r}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \dot{\boldsymbol{\omega}} \times \mathbf{r} = \nabla U \quad (5)$$

and the equations in scalar form are provided below.

$$\begin{aligned} \ddot{x} - 2\omega \dot{y} &= \omega^2 x - U_x \\ \ddot{y} + 2\omega \dot{x} &= \omega^2 y + U_y \\ \ddot{z} &= U_z \end{aligned} \quad (6)$$

The spacecraft's position and velocity is denoted by $\mathbf{r} = [x, y, z]$ and $\mathbf{v} = [\dot{x}, \dot{y}, \dot{z}]$ respectively are represented in the body-fixed frame centered on the asteroid.

2.3 Zero-velocity surface

In the body-fixed reference frame, the gravitational potential field is time invariant and the rotational rate of an asteroid is constant (i.e. $\dot{\boldsymbol{\omega}} = 0$), thus the equations of motion (eq. (6)) is time invariant. There exists an integral of motion, a conserved quantity, known as the Jacobi integral [10], it is constant for all motion and is defined in eq. (7).

$$J = \frac{1}{2}\omega^2(x^2 + y^2) + U(r) - \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) \quad (7)$$

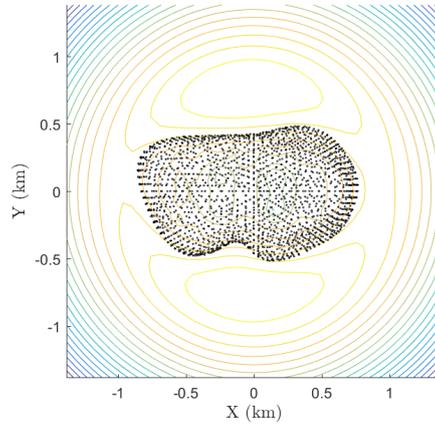


Fig. 3: Zero Velocity curve of Castalia (XY-plane)

The existence of the Jacobi integral allows us to define the stable regions, equilibrium points and zero velocity surfaces around the asteroid [10]. The computed zero velocity curve of the asteroid Castalia is shown in fig. 3. Four equilibrium points exists around the asteroid Castalia and can be found directly by identifying the points where the curve intersects or bifurcate. All four synchronous orbits around asteroid Castalia are found to be unstable [10].

3. Simulation and Results

In this section, numerical simulations of the periodic, quasi-periodic and terminator orbits are presented. For each simulated case, a separate analysis of minimum close approach window between the orbiting master spacecraft and CubeSat is undertaken. The difference in velocity, Δv , between two satellites should also be minimized such that the master spacecraft spends minimal energy on a manoeuvre to capture and dock orbiting CubeSat. The Δv requirement is set to be comparable to hovering operations limits in previous missions and it varies from 5~10 cm/s per day [11]. In each of the simulated orbital conditions, we analyse the results based on both the close approach time window and the minimum Δv required.

3.1 Periodic orbits

The properties of a trajectory near an asteroid change with the distance from the centre of the asteroid. Orbits within a few asteroid radii (depending on asteroid's size) are dominated by the asteroid's gravity attraction and other perturbations can be neglected. Considering the non-uniform gravity potential, finding stable *periodic orbits* in the vicinity of the asteroid is critical for the mission. One well known approach is the *monodromy method* [12] which starts with an approximate periodic orbit and uses an iterative estimation approach (such as Newton-Rhapson) to close-in on the exact periodic orbit. The computed stable periodic equatorial orbit using monodromy method is shown in fig. 4

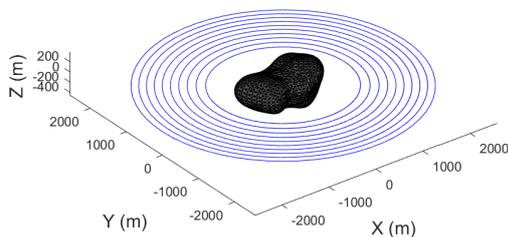
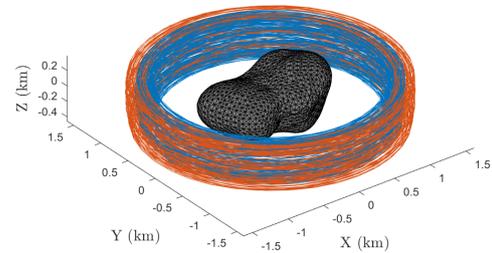
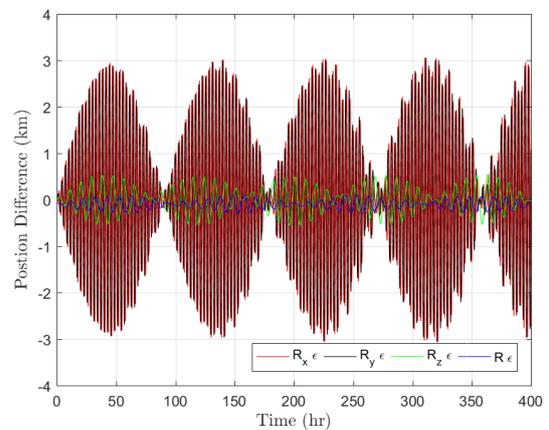


Fig. 4: Stable periodic orbits around Castalia

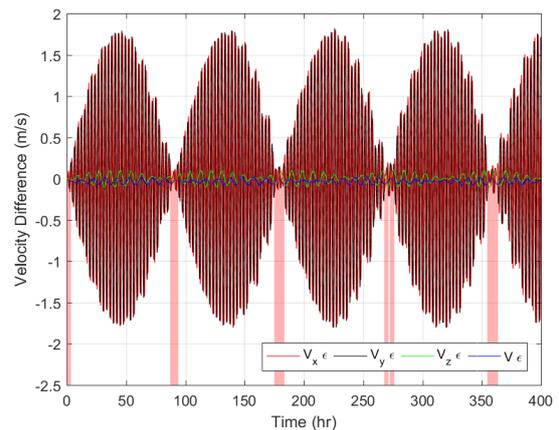
In the periodic orbit simulation, the master spacecraft and CubeSat are placed in an orbit inclined at an angle of 10° . The gravitational potential of the asteroid is considered for this sim-



(a) Orbit manifold



(b) Position difference between two satellites



(c) Δv between two satellites and the band highlights the minimum 12 hr window with $\Delta V < 20$ cm/s

Fig. 5: Both satellites in inclined periodic orbits

ulation and the solar perturbation is neglected as shown in eq. (6). The orbit of both satellites are propagated for a total time period of 400 hours, the initial position (km) and velocity (m/s) of two satellites are $[1.3704, 0, -0.2335, 0, -0.8756, 0]$, $[1.2879, 0, -0.2178, 0, -0.8500, 0]$ and the results are shown in fig. 5. As highlighted in fig. 5c, for every 100 hours there exists a close approach window of 12 hours minimum and the required Δv between the two orbits is less than 20 cm/s. The position difference between the satellites are well separated from each other most of time. If the Δv limit is reduced to 10 cm/s, the close approach window duration is severely limited. This suggests that the combinations of periodic orbits are suitable to have a reusable relay platform around the asteroid, but the required Δv is double than the considered threshold.

3.2 Quasi-periodic orbits

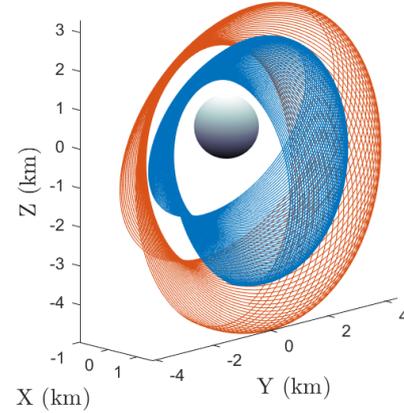
As the orbital altitude of a spacecraft increases, the influence of the Sun's gravity and SRP becomes significant on the dynamics. These perturbations cause the orbital plane to precess over time and this family of orbits are named as *quasi-periodic orbits*. An orbit is quasiperiodic if it contains only a finite number of incommensurate frequencies [13]. The coordinate frame is defined such that \hat{x} direction points from the sun to the asteroid, \hat{z} direction aligned with the angular velocity of the asteroid and \hat{y} direction complete a right hand system. The equations of Augmented Normalized Hill Three Body Problem (ANH3BP) provided in eq. (8), are used to describe the spacecraft motion near the small celestial under the influence of SRP ([14], [15]).

$$\begin{aligned}\ddot{x} &= 2\dot{y} + 3x - x/\|\mathbf{r}\|^3 + \beta \\ \ddot{y} &= -2\dot{x} - y/\|\mathbf{r}\|^3 \\ \ddot{z} &= -z - z/\|\mathbf{r}\|^3\end{aligned}\quad (8)$$

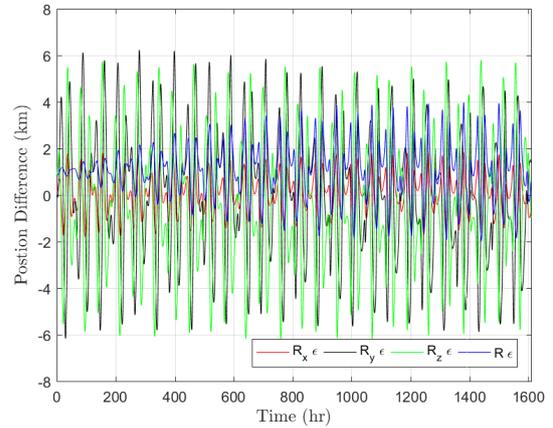
In eq. (8), the non-dimensional acceleration β is given by

$$\beta = \frac{G1}{(m/A)\mu_{Sun}^{2/3}\mu_{Ast}^{1/3}}\quad (9)$$

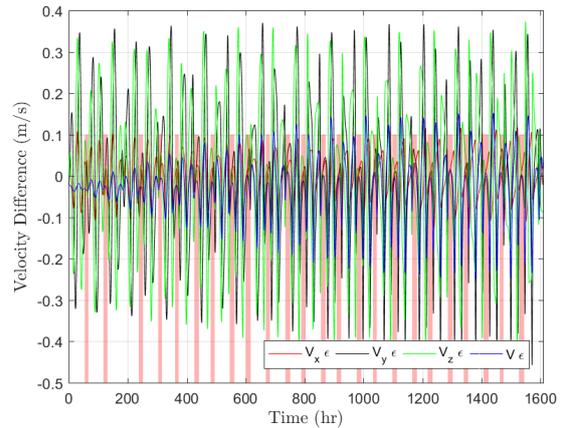
For the normalization, the unit length $(\mu_{Ast}/\mu_{Sun})^{1/3}.R$ and the unit time is $1/N$ where $N = \sqrt{\mu_{Sun}/R^3}$ is the mean motion of the asteroid orbit. Since we are analyzing an orbit around asteroid Castalia, unit length is found to be 146.68 km and unit time is 67.05 days, β is 16.9. In this case the master satellite is assumed to be in a terminator orbit and the CubeSat's orbit is inclined 70° with the orbit normal facing towards the Sun. The simulated



(a) Orbit manifold



(b) Position difference between two satellites



(c) Δv between two satellites and the band highlights the minimum 12 hr window with $\Delta V < 10$ cm/s

Fig. 6: Master satellite in terminator orbit and CubeSat in an inclined orbit

orbit results for unit time are presented in fig. 6. As highlighted in fig. 6c there exists multiple close approach window of 12 hours minimum and the required Δv between the two orbits is less than 10cm/s. The position difference also suggest that the satellites are well separated from each other most of time and thus making at least one satellite available to the rover on the asteroid surface. This suggests even if the CubeSat is placed in an inclined orbit and master satellite in a terminator orbit, it is possible to make close approach manoeuvre and capture the orbiting CubeSat.

3.3 Quasi-terminator orbits

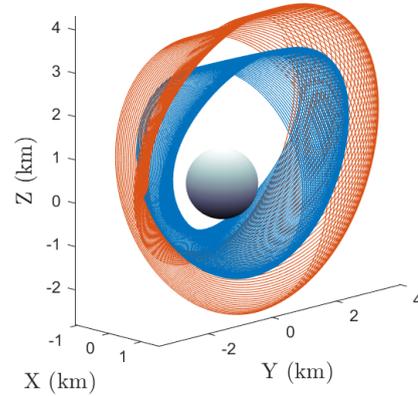
When combining the non-uniform gravity potential with the solar radiation perturbations any motion in the asteroid orbital plane will tend to increase eccentricity. This effect tends to reduce as the spacecraft's orbit inclination increases and it will reach a minimum, when the orbital plane is inclined to 90° and orients with the sun-terminator plane [16]. These orbits are known as *terminator orbits* and the orbits precess due to SRP are also known as *quasi-terminator orbits*. These orbits are a family of quasi-periodic orbits, they are relatively stable and do not need station keeping maneuvers.

In this case, both master spacecraft and CubeSat are assumed to be orbiting in two separate terminator planes and with different semi-major axes. The initial position (km) and velocity (m/s) of two satellites are [0.9, 0, -2, 0, 0.2168, 0] and [1, 0, -2.8, 0, 0.1833, 0] and the orbits are simulated for unit time \sim 1600 hours.

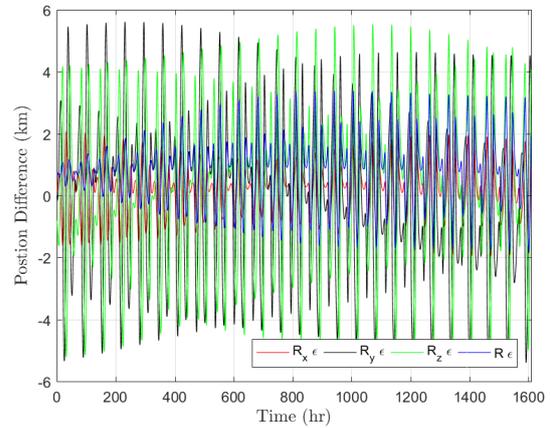
As seen in the fig. 7 there exists a periodic approach window of approx. 12hr for the Master satellite to capture the CubeSat if needed. In this case the Δv required for the two orbits is similar (fig. 7c) to the quasi-periodic but, it is found to more stable as shown in fig. 8 and fig. 9. Also, they achieve a periodic close approach window length of nearly 12 hours. This suggests that quasi-terminator orbits are more suitable orbits with uniform distribution of close approach window and requires a smaller Δv . As the orbit precess over the sun-lit side larger area of the asteroid will be covered and thus making it possible to relay the communications to the rover from the master spacecraft.

4. Conclusion

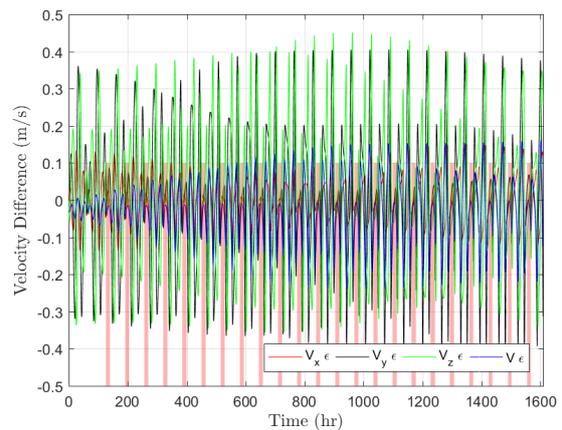
This work analyzes multiple orbital configurations for a generic asteroid mission architecture comprised of a master spacecraft and CubeSat communications relay to support a lander/rover. The assumptions in-



(a) Orbit manifold



(b) Position difference between two satellites



(c) Δv between two satellites and the band highlights the minimum 12 hr window with $\Delta V < 10$ cm/s

Fig. 7: Both satellites in terminator orbits

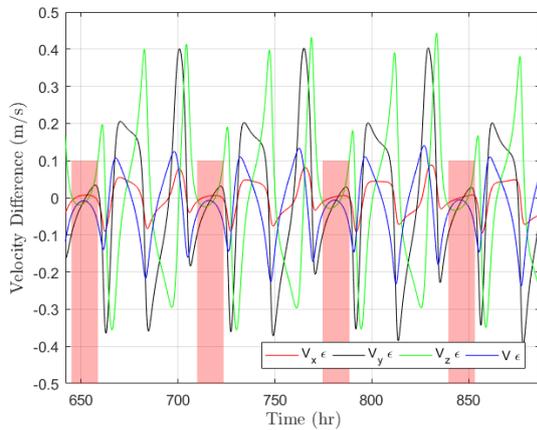


Fig. 8: Δv and highlighted close approach window in quasi-terminator orbits

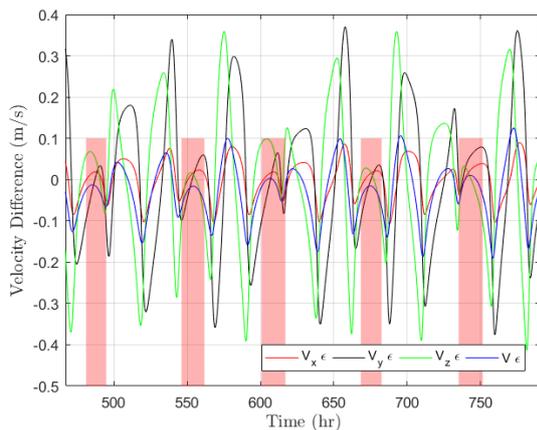


Fig. 9: Δv and highlighted close approach window in quasi-periodic orbits

clude the CubeSat platform require constant docking with the master spacecraft and hence the orbital analysis was focused on the close approach time window between the two trajectories to achieve the above mentioned objective. Trajectory combinations selected from a general family of orbits around the asteroids. Simulations presented for three specific cases of orbits including periodic, quasi-periodic and quasi-terminator orbits. The close approach window of 12 hours minimum between the two trajectories is found to be uniformly distributed in all three cases. In quasi-terminator orbits the Δv is more stable than the trajectories in other two cases. This suggests that the quasi-terminator orbits are relatively better option of the three simulated cases.

This work represents a conceptual study and in future a more detailed analysis of the orbits is required based on the actual mission scenarios. Future scope of this work can be extended to analysing the availability of relay platforms to the rovers over a specific time period and grid search to find other possible orbit combinations. Considering the challenges in orbital rendezvous and docking, detailed analysis of the close approach time window and the Δv limits is required for a careful analysis based on the prospective mission scenarios. Since CubeSat technologies are evolving, there are also possibility of more modular platform will be available in future missions capable of helping the rovers with localization and navigation and possibly autonomous swarms will help mining the asteroids.

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